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The Importance of Quantum Effects in Superconducting Cosmic Strings

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Classical current-carrying cosmic string loop motion is investigated by means of a numerical simulation making explicit use of the Carter formalism and the Carter–Peter rational and logarithmic equations of state. The class of initial configurations consists of elliptic loops far from the vorton equilibrium state with constant state parameter along the loop. Thus, the relevant parameters, apart from those describing the equation of state itself, are the ellipticity and the initial state parameter. It is found that for most of the parameter space, the loop motion is quasiperiodic, but that this result is actually irrelevant to the treatment of an arbitrary loop motion: indeed, almost systematically, the loops develop kinks and cusps, and in the case of spacelike currents, there are segments of the loop that escape the elastic regime. It is then argued that quantum effects resulting from these situations will in practice provide the dominant evolution mechanisms.

1. INTRODUCTION

Cosmic strings [2, 3] have been proposed as a possible means of forming large-scale structures in the universe, yielding at the same time an imprint in the cosmic microwave background (CMB). Their distribution was computed numerically [4] and it was shown that it consists of mainly (roughly 80%) a scale-invariant distribution of loops together with "long strings," i.e., loops whose characteristic size exceed that of the observable universe. These results were derived using the simplest field model that leads to the existence of cosmic strings, namely the Abelian Higgs model. As a result, the strings are structureless, so that the only nonvanishing components of their stress-energy tensors, the energy per unit length U and tension T, are equal and constant. Once approximated as a two-dimensional worldsheet, the string dynamics is

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then simply derivable from the Goto–Nambu action [5], i.e., the area spanned by the string.

In 1985, Witten [6] made the point that when one considers a realistic field model, the Higgs field is very often coupled to other fields, in particular fermions and gauge fields, to which it gives masses (as was the original aim of the Higgs mechanism). He realized that these couplings might imply the possibility that some particles get trapped in the string cores, thereby potentially supporting currents. In the special case where the current was coupled to electromagnetism, he showed that its intensity was proportional to the time integral of an applied external electric field. In other words, the string behaves as a superconducting wire (with some differences, however, as the extreme thinness of these strings prevents the Meissner effect, for instance, from taking place); Witten called them superconducting cosmic strings. It is now believed that most of the particle physics models having strings predict them to be superconducting.

Meanwhile, a formalism aiming at a description of a p-dimensional object living in an n-dimensional manifold was set up by B. Carter, making use of the equation of state relating the energy per unit length and the tension [7, 8]. In this formalism, one needs an extra parameter, called the *state parameter*, and interpretable as the square of the phase gradient of an effective scalar field defined on the string itself. Therefore, this formalism provides a tool to examine the dynamics in particular of a superconducting cosmic string, which, contrary to the Goto–Nambu string, is endowed with a rich structure. For instance, the amplitude of the current, that may vary along the string, can be chosen as the state parameter.

The actual microscopic structure of these strings was then elucidated in a series of works [9–12] until an equation of state was derived analytically [13], based on their known properties, such as, in particular, the current saturation (or quenching [14]) and the phase frequency threshold [10] (implying a divergence of the charge per unit length when the state parameter approaches the mass of the trapped particle). This was derived for a bosonic condensate and used extensively afterward under the assumption, emphasized by Witten, that the string worldsheet being two-dimensional, even a fermionic current was essentially describable by means of a scalar field. Work is currently in progress to calculate the effective equation of state for a four-dimensional microscopic string model having fermionic carriers [15].

Solutions describing the motion of a Goto–Nambu string were obtained [16] and it was shown that most initial conditions yield the existence of transient phenomena occurring along the string, called kinks and cusps, i.e., regions of the string where the curvature can become sufficiently high that quantum effects are expected to be dominant. As is shown here, this can be generalized to the superconducting case, with a difference as far as the conclusions are concerned: a Goto–Nambu loop is doomed to decay into a bunch of Higgs and gauge vector boson radiation, so taking account of the presence of kinks and cusps does not really modify the status of a string network for which the disappearance of loops is already taken into account. Superconducting strings, on the other hand, have a nondegenerate equation of state, i.e., the energy per unit length is different from the tension. Thus, the Lorentz invariance along the string that exists in the Goto–Nambu case is violated by the very existence of the current. As a result, a loop can be assigned a physically measurable angular momentum; in other words, the tension that tends to make the loop shrink can be balanced by the centrifugal force due to rotation in such a way as to yield a new equilibrium configuration, called a vorton [17].

The question of whether vortons form and if so if they are stable is of uttermost importance since it was evaluated that they would lead to a cosmological catastrophe [18]: indeed, as the remnant density of loops would scale as matter (Goto–Nambu loops, because they decay quite rapidly, yield an energy density in the universe that ultimately scales like radiation), very massive vortons would easily dominate the evolution of the universe. In fact, considering cosmic strings formed at the grand unified (GUT) phase transition reproduces essentially the monopole problem.

Until now, the problem has been studied at two different levels, namely that of the classical stability of vortons, which has been examined in general [19, 20] and established in the particular case of Witten superconducting loops [21], and the approach to equilibrium [8, 22]. The latter was investigated by considering an already circular loop, for which it was shown that it evolves in a self-potential depending only on its radius and on the initial state parameter. Quantum stability has been briefly looked at [23], and this subject was essentially neglected; it will be soon clarified with the emergence of new quantum calculations [15].

All this leaves one stage to progress further: starting with an arbitrary shaped loop, one needs decide if a vorton might form. This is the point we intend to treat here by means of the classical equations of motion in the thin-string limit, making use of the Carter–Peter equation of state. The next section recaps the relevant formalism and exhibits the two-dimensional equations of motion (we have assumed the loop to remain in a plane to begin with). Then we go on to the actual numerical simulation: as the equations are highly nonlinear, no analytic solution could be found except the circular equilibrium ones. Those are used as a means of checking the numerics.

We have decided to study a class of solutions that consist of initial configurations having elliptic shape with arbitrary (and constant) state parameter. Therefore, we have a two-dimensional parameter space to analyze, once the ellipticity is taken into account. What was found, contrary to our expectations, is that not only do kinks and cusps very often develop, but also some regions of the loops show a tendency to move out of the elastic regime where the thin-string formalism is valid. In both cases, quantum effects will become dominant. Further work is needed to incorporate these effects into the simulations.

2. GENERAL EQUATIONS OF MOTION

A superconducting cosmic string can be microscopically described through the condensation of a complex scalar field $\Sigma(x^\mu)$ whose phase may vary along the string worldsheet. Explicitly, this can be examplified by considering locally a piece of string as an infinite straight string lying an axis z, giving the ansatz, using cylindrical coordinates,

$$\Sigma(x^\mu) = \sigma(r) \exp [i(\omega t - kz)] \tag{1}$$

so that the state parameter is

$$w = k^2 - \omega^2 \tag{2}$$

whose sign therefore reflects the timelike ($w < 0$) or spacelike ($w > 0$) character of the corresponding current [10].

When one considers an arbitrary shape, one needs two internal string coordinates ξ^a, with the identification $\xi^0 \equiv \tau = t$ and $\xi^1 \equiv \ell = z$ no longer automatically feasible (although choosing the proper time along the worldsheet to be the coordinate time is always possible and amounts just to a gauge choice). The string location is then the set $\{x^\alpha(\xi^a)\}$ thanks to which an internal metric γ_{ab} can be defined

$$\gamma_{ab} = g_{\mu\nu} \frac{\partial x^\mu}{\partial \xi^a} \frac{\partial x^\nu}{\partial \xi^b} \tag{3}$$

with inverse γ^{ab}. In these general conditions, the state parameter reads

$$w \equiv \kappa_0 \gamma^{ab} \partial_a \psi \partial_b \psi \tag{4}$$

where now the scalar condensate reads

$$\Sigma(x^\mu) = \sigma(x_\perp) \exp[i\psi(\xi^a)] \tag{5}$$

The set $\{x_\perp\}$ represents the coordinates orthogonal to the string worldsheet. The coefficient κ_0 is a normalization factor.

Carter's formalism states that the knowledge of a Lagrangian function $\mathcal{L}(w)$ is enough to entirely determine the string dynamics, provided it is not coupled to an external long-range field such as electromagnetism, using the action

$$S = \int d^2\xi \ \sqrt{-\gamma}\mathcal{L}(w) \tag{6}$$

where γ is the determinant of the induced metric γ_{ab}. Varying this action with respect to the variables $x^\alpha(\xi^a)$ yields the stress-energy tensor conservation

$$\eta^\rho_\mu \nabla_\rho \bar{T}^{\mu\nu} = 0 \tag{7}$$

the latter being defined as usual through

$$\hat{T}^{\mu\nu} \equiv \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}} \tag{8}$$

and

$$\sqrt{-g}\hat{T}^{\mu\nu} = \int d^2\xi \ \sqrt{-\gamma} \ \bar{T}^{\mu\nu} \ \delta^{(4)} \left[x^\rho - x^\rho(\xi^a)\right] \tag{9}$$

and the first fundamental tensor of the string worldsheet being

$$\eta^{\mu\nu} = \gamma^{ab} \frac{\partial x^\mu}{\partial \xi^a} \frac{\partial x^\nu}{\partial \xi^b} \tag{10}$$

As it turns out, the stress-energy tensor may be cast in the form

$$\bar{T}^{\mu\nu} = U(w)\, u^\mu u^\nu - T(w)\, v^\mu v^\nu \tag{11}$$

where u^μ and v^μ are respectively a timelike and spacelike unit vector tangent to the wordlsheet. The energy per unit length U and the tension T can both be calculated knowing only the Lagrangian function $\mathcal{L}$ (see ref. 8 for details): defining

$$\mathcal{K} \equiv \left(-2\frac{d\mathcal{L}}{dw}\right)^{-1} \tag{12}$$

and

$$\chi = w\mathcal{K}^{-2} \tag{13}$$

(χ being interpretable as the current flowing along the string), one can obtain another formulation in terms of a master function $\Lambda(\chi)$ related to the Lagrangian function by means of the Legendre transformation

$$\Lambda = \mathcal{L} + \mathcal{K}_\chi \tag{14}$$

This is another formulation of the same physics, as it can be shown [8] that varying the action $\int d^2\xi \ \sqrt{-\gamma}\Lambda(\chi)$ with respect to $x^\alpha(\xi^a)$ yields the same dynamics as varying the action (6). For $w > 0$, i.e., for a spacelike current, the energy per unit length turns out to be identifiable with the Lagrangian

($U = -\mathcal{L}$) and the tension with the master function ($T = -\Lambda$), while for $w < 0$, i.e., a timelike current, the inverse relations $U = -\Lambda$ and $T = -\mathcal{L}$ hold.

We restricted our attention to a two-dimensional situation where a string loop lies on a (x, y) plane and we fixed the gauge to be that for which the proper time τ is identified with the coordinate time t, while the space coordinate ℓ is the (conveniently rescalled) phase of the bosonic condensate ψ. Thus, the interesting set of unknown functions to be dynamically determined is

$$x^{\alpha}(\xi^{a}) \equiv [t, x(t, \psi), y(t, \psi)] \tag{15}$$

Setting

$$\beta = \dot{x}x' + \dot{y}y' \tag{16}$$

$$\Delta = \dot{x}y' - \dot{y}x' \tag{17}$$

$$n^2 = x'^2 + y'^2 - \Delta^2 \tag{18}$$

$$\dot{z}^2 = 1 - \dot{x}^2 - \dot{y}^2 \tag{19}$$

one finds the state parameter as

$$w = \pm \dot{z}^2 / n^2 \tag{20}$$

where the sign must be chosen according to whether the current is required to be timelike or spacelike. Dots and primes respectively denote derivatives with respect to t and ψ. Using these notations, we find the equations of motion

$$\ddot{x} = \frac{Ax' + B(y' - \dot{x}\Delta)}{y'(y' - \dot{x}\Delta) + x'(x' + \dot{y}\Delta)} \tag{21}$$

and

$$\ddot{y} = \frac{Ay' - B(x' + \dot{y}\Delta)}{y'(y' - \dot{x}\Delta) + x'(x' + \dot{y}\Delta)} \tag{22}$$

where

$$A = \frac{\dot{z}^2 c_L^2}{n^2 - \beta^2 c_L^2}\left[\dot{z}^2(c_1 x'' + c_2 y'') + 2\beta(c_1 \dot{x}' + c_2 \dot{y}')\right] \tag{23}$$

$$B = \frac{\dot{z}^2 c_T^2}{n^2 - \beta^2 c_T^2}\left[\dot{z}^2(y' x'' + x' y'') + 2\beta(y' \dot{x}' - x' \dot{y}')\right] \tag{24}$$

$$c_1 = x' + \dot{y}\Delta \tag{25}$$

$$c_2 = y' - \dot{x}\Delta \tag{26}$$

and the transverse and longitudinal velocities are defined as

$$c_T^2 = \frac{T}{U} \tag{27}$$

and

$$c_L^2 = -\frac{dT}{dU} \tag{28}$$

Solving these equations is the purpose of the following section, but for now, let us turn to the equation of state.

3. EQUATION OF STATE

The microscopic structure of a superconducting cosmic string involves two mass scales, namely m, the energy scale of the symmetry breaking responsible for the very existence of the cosmic strings themselves, and m_*, essentially the mass of the trapped particles. Detailed examination of the field equations [9, 10] revealed that two phenomena take place: first, the amplitude of a spacelike current happens to be limited (current quenching). This nonlinear effect is interpretable in the fermionic case by saying that a particle momentum cannot exceed its Fermi level, so that any attempt to add extra particles yields unstable states. On the other hand, for timelike currents, the energy of the bound particles must be less than their mass, otherwise they become free particles; this implies a divergence in the current which was evaluated as a single pole in the function $\mathcal{K}^{-1}$.

Taking the pole into account is easily done by considering a logarithmic model with

$$\mathcal{L}(w) = -m^2 - \frac{m_*^2}{2} \ln \left\{ 1 + \frac{w}{m_*^2} \right\} \tag{29}$$

and the only relevant dimensionless parameter is

$$\alpha \equiv \left(\frac{m}{m_*} \right)^2 \tag{30}$$

Once every quantity has been rescalled in terms of m, the numerical calculations can be performed using α as unique underlying parameter. This Lagrangian reproduces the numerically derived equation of state for a field model in the electric regime very accurately. However, the fit is not so good in the magnetic regime, and it was found that a much better accuracy was obtained by the so-called rational model with Lagrangian function given by

$$\mathcal{L}(w) = -m^2 + \frac{w}{2}\left\{1 - \frac{w}{m_*^2}\right\}^{-1} \tag{31}$$

This model was therefore used in the case where w was set to positive values.

A sketch of the equation of state for a specific bosonic carrier superconducting string model is shown in Fig. 1.

4. NUMERICAL RESULTS

We have run a simulation to solve Eqs. (21) and (22) for the class of initial conditions consisting of elliptic loops with constant state parameter allowed to vary in all its available range defined by the conditions $c_L^2 > 0$ and $c_T^2 > 0$. Figure 2 shows a typical configuration in the (x, y) plane together with the velocities (the value of the state parameter fixes them unambiguously). Circular loops with vanishing ellipticity $e = 0$ have been used as a numerical check of the stability of the code: as they should evolve in a self-potential depending only on their radius, the comparison can be achieved quite easily. Besides, for a rotation velocity equal to c_T, we know that one must have a vorton state, i.e., an equilibrium configuration. This also can be used as a means of checking the accuracy of the code.

The precision was estimated using the total energy of the loop

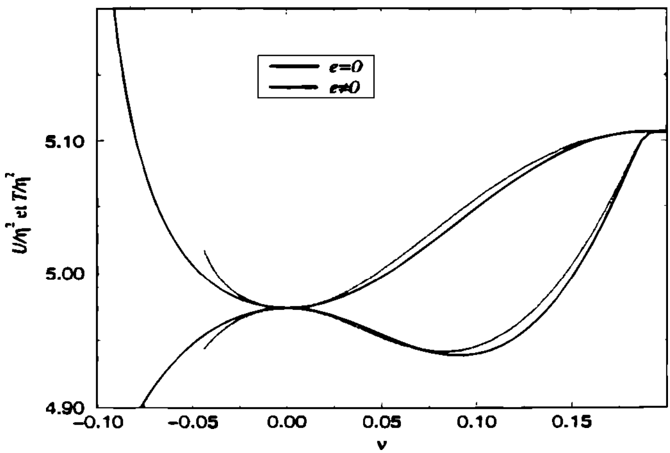

Fig. 1. Typical equation of state for the Witten bosonic model. The energy per unit length U and tension T are shown as functions of the square root of the state parameter $\nu = \text{Sign}(w)|w|^{1/2}$. The saturation of the current appears on the r.h.s of the plot, while the charge divergence is seen on the l.h.s. Also shown is the inclusion of electromagnetic corrections in the equation of state when the carrier is considered coupled to an external electromagnetic field. Those corrections are seen to be mostly negligible in the range of interest.

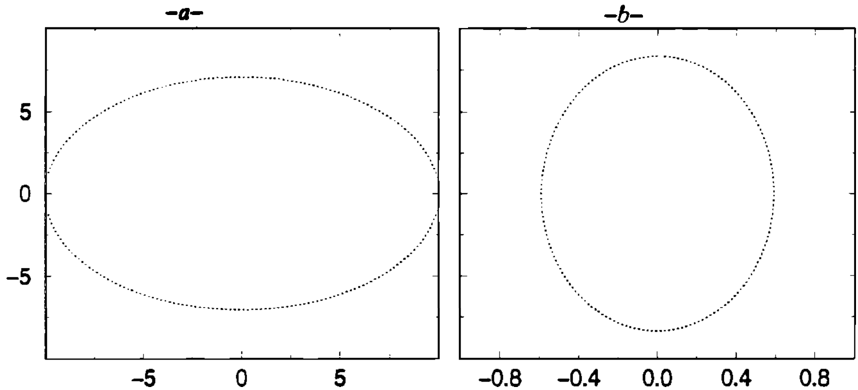

Fig. 2. Initial configuration for a magnetic elliptic loop with semimajor axis $a = 10$ (in units of m^{-1}). Parameters are $m_* = m$ and $\nu_0 = 3/4\ m_*$. The dotted line is for points where $x^2 < c_L^2$, and the full line for the opposite. (a) The coordinates x and y of the initial string worldsheet; (b) the velocities as measured by x and y.

$$E = \int d\psi\ U(\psi) \frac{n^2 - \beta^2 c_T^2}{n\dot{z}^2} \tag{32}$$

which should be conserved since the loop is isolated and not coupled to any radiation field. We imposed a precision of $|\Delta E/E| < 10^{-6}$ over an integration time $T \sim 1000 m^{-1}$; most of the interesting effects take place long before this time is reached.

4.1. The Magnetic Regime

A typical elliptic loop evolution is shown for a succession of times in Fig. 3. Over the range of parameter studied, this is a generic situation: the elliptic loop starts rotating until it gets deformed in shape while slightly shrinking. Almost independently of the initial parameters used in the simulation, it reaches a point where an even number of regions become unstable with respect to longitudinal perturbations, i.e., the current enters a stage where it should have quenched already. In other words, the string is oversaturated. This means in practice that it has left the elastic regime for which the thin-string description is valid. A more appropriate field treatment, i.e., not neglecting the thickness of the torus, should be used. Such a simulation is beyond the reach of the present work, but can presumably be taken differently into account. I shall turn to that point in the conclusions.

4.2. The Electric Regime

The typical evolution of an elliptic loop in the electric regime is shown in Fig. 4; it also occurs in some cases for the magnetic regime. In this case,

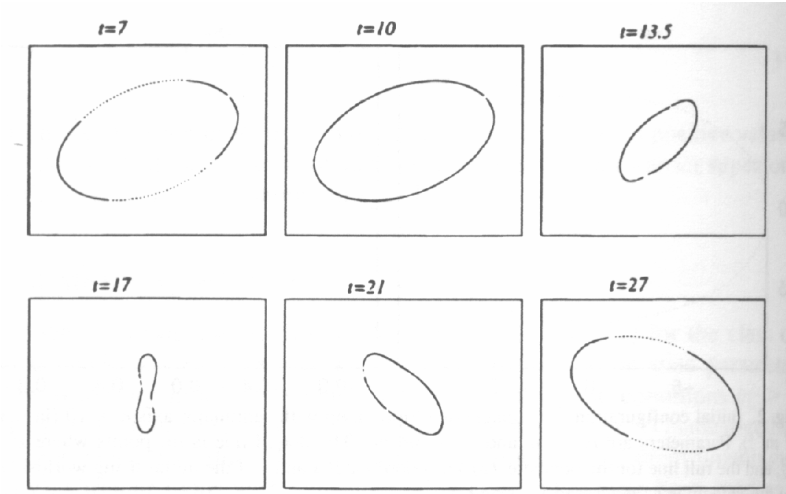

Fig. 3. Evolution of a magnetic string elliptic loop with semimajor axis $a = 10$. Both lengths and time are in units of m^{-1}. The parameters have been fixed to those of Fig. 2. Here the dotted line shows the point along the string where $x^2 < c_L^2$, the full line means the opposite, and a dashed line represents points for which $c_L^2 < 0$ (see $t = 17$) and where, therefore, longitudinal instabilities followed by massive quantum radiation are expected to occur. As a result, all the simulation for times $t < 17$ are presumably not physical. Essentially similar results hold for electric strings, leading, however, in this case to the appearance of points where $c_T^2 < 0$.

the loop remains everywhere in the elastic regime so that the thin-string approximation could seem to be valid all along. However, this is not the case, as some regions get tiny curvature radius or even should reconnect, a fact that is not accounted for in the code. More work is needed in the last case to assess whether this latter possibility is not just an artifact of working in two dimensions; however, that would not modify the former. Hence the conclusion in that case would be that, here again, a full field simulation should be done as the curvature radius may be as small as the actual thickness of the string.

5. CONCLUSIONS

A-two dimensional simulation of the evolution of superconducting cosmic string loops has been presented using the Carter formalism and both the rational and logarithmic equations of state describing respectively spacelike and timelike currents. Nonperturbative effects have been taken into account by considering initial configurations far from the equilibrium vorton state. Two questions were at the origin of this work, namely that of vorton formation, i.e., Given an arbitrary shaped loop, what is the probability that it ends up

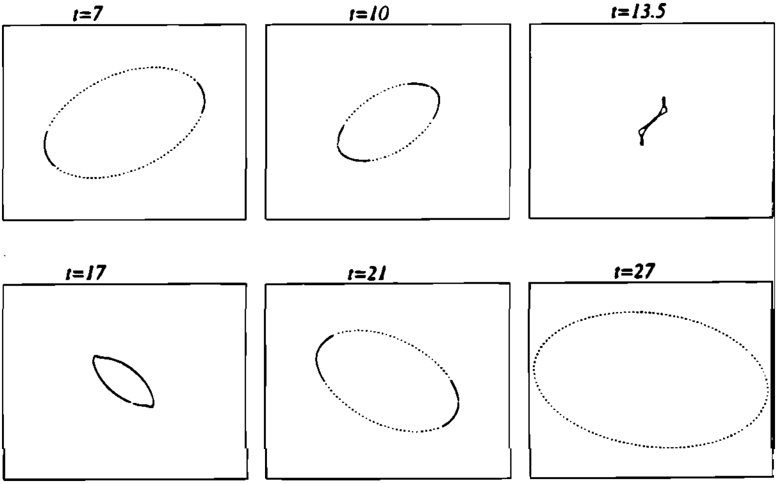

Fig. 4. Evolution of a magnetic or electric string elliptic loop with semimajor axis $a = 10$ in a characteristic situation where the loop always remains everywhere in the elastic regime. Both lengths and time are in units of m^{-1} and the configuration shown here is electric. The parameters have been fixed to those of Fig. 2 except that $\nu_0 = 1.75 m_*$. The initial configuration actually looks completely similar to Fig. 2. Conventions are those of Fig. 3. It is seen here that even though the loop always remains in the elastic regime, regions of very high curvature radius (particularly in $t = 13.5$) form that will be responsible for massive radiation.

in a vorton state? and also, What is the fate of unstable perturbations? These two questions yield another quest that should now be tackled.

The typical evolution of a loop reveals that it will either exit the elastic regime, or it will bend so much on itself that its curvature radius might locally become less than its thickness. In both cases, the thin-string approximation ceases to be valid, although for different reasons. In practice, that means that quantum effects will be dominant in these regions so that the simulation, once these regions have been formed, is presumably physically unreliable. We do not believe this to be an artifact of the restricted two-dimensional analysis.

Whenever quantum effects start to dominate, we can guess what the actual mechanisms will be, and they will consist in ejection of trapped particles outside the string core. In the magnetic regime, this effect will tend to reduce the amplitude of the current, and thus help the string to get back to the elastic regime. For the electric case, on the contrary, the state parameter will be reduced (this is, by the way, another illustration of the dual formalism) so that, as can be seen in Fig. 1, that will increase the tension. Therefore, the string will be less bent. Here again, the thin-string approximation might be recovered. So the problem now reduces to that of obtaining an effective action that would take the carrier radiation into account. At the time of writing, there is no such formalism available, although it is currently under construction.

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